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Disclaimer

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A spectral subgrid-scale eddy viscosity and magnetic resisitivity model based on the eddy-damped quasi-normal Markovian (EDQNM) spectral kinetic and magnetic energy transfer presented in [12] is used in large-eddy simulation (LES) of large kinetic and magnetic Reynolds number magnetohydrodynamic (MHD) turbulence. The proposed model is assessed via *a posteriori* tests on three-dimensional, incompressible, isotropic, non-helical, freely-decaying MHD turbulence at asymptotically large Reynolds numbers. Using LES with an initial condition characterized by an Alfvén ratio of kinetic to magnetic energy r_A equal to unity, it is shown that the kinetic energy spectrum $E_K(k)$ and magnetic energy spectrum $E_M(k)$ exhibit Kolmogorov $-5/3$ inertial subrange scalings in the LES, consistent with the EDQNM model.

PACS numbers:

Numerical simulations for investigating the physics of magnetohydrodynamic (MHD) turbulence are of the greatest interest. As most astrophysical and geophysical plasmas (e.g. the liquid core of the Earth, accretion disks, and star-forming molecular clouds) cannot be directly investigated experimentally, numerical simulations are the method of choice for studying the properties of such plasmas. In the field of applied and fundamental research related to engineering and physical sciences, most flows are highly turbulent. Even with contemporary supercomputing capability, direct numerical simulation (DNS) remains limited to turbulent flows with modest Reynolds numbers. Large-eddy simulation (LES) overcomes this computational limitation by computing only the largest resolved scales and modeling the effects of the subgrid scales on these resolved scales using physical arguments to approximate turbulent energy dissipation and backscatter [7].

LES is based on the separation of the spatial scales of motion: the resolved scales are directly computed by solving partial differential equations consisting of the usual governing equations supplemented with a subgrid-scale term which account for the effects of the unresolved scales of motion. Two different types of subgrid-scale models can be used for isotropic turbulence: physical space models for finite-element, finite-volume and finite-difference methods, or spectral models for (pseudo)-spectral methods. Physical space subgrid-scale models for MHD flows are typically extensions of the models introduced in the pure hydrodynamic case, but are used along with non-spectral numerical methods which are less accurate for the resolution of the small scales than spectral methods. The numerical error induced by these schemes is often very large in the vicinity of the cutoff wave number, resulting in a poor estimate of the energy transfer across the cutoff scale. On the other hand, spectral methods are more accurate at the small scales where the interscale dynamics can be well captured, yielding an accurate prediction of the subgrid-scale energy transfer. This remark is well founded, as shown by the energy transfer cusp, observed near the cutoff in direct numerical simulations. This cusp is underestimated in physical space eddy viscosity-type subgrid models with model constants computed dynamically, as shown in [4] by comparison with filtered DNS data, whereas it is clearly present in the spectral subgrid model as shown, for example, in the small Prandtl number MHD subgrid-scale model of Ponty *et al.* [6].

Historically, subgrid-scale models for MHD turbulence expressed in physical space have been constructed by extending the usual non-magnetic models to the case of electrically-conducting fluids [11]. MHD gradient-diffusion type subgrid-scale models have been assessed for isotropic turbulence at magnetic Prandtl number $Pr_m = 1$ [1, 4] and at small magnetic Prandtl number [3]. In the anisotropic case, this type of subgrid-scale model was used for turbulent channel flow at small magnetic Prandtl number and kinetic Reynolds number $Re = 29000$ [10]. Additionally, a physical space subgrid-scale model based on the ideal cross-helicity invariant of

MHD turbulence has been developed [4, 5] to account for the cross-helicity inverse cascade in the energy transfer. These models appear to capture the principal features of incompressible homogeneous MHD turbulence at a computational cost orders of magnitude lower than a DNS.

More recently, a spectral subgrid-scale model was developed for MHD turbulence [12] using an analysis of the eddy-damped quasi-normal Markovian (EDQNM) subgrid-scale kinetic and magnetic energy transfers in isotropic turbulence; however, this model has not yet been assessed using LES. This model was derived by applying a sharp Fourier cutoff filter to the spectral energy transfer equations, generalizing the analysis in [8] to MHD turbulence. In the case of small magnetic Prandtl number statistically-stationary turbulence with an external constant large-scale magnetic field, Ponty *et al.* [6] empirically constructed a Chollet-Lesieur [2] type spectral subgrid model. In these small magnetic Prandtl number simulations, the magnetic field fluctuations are fully resolved and the subgrid velocity fluctuations are modeled using LES. The results were found to be in good agreement with existing experimental data for large kinetic Reynolds number.

In nondimensional form, the unforced incompressible MHD equations (with unit constant density) are written as

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} &= (\nabla\times\mathbf{b})\times\mathbf{b} - \nabla p + \nu_T\,\nabla^2\mathbf{v} \quad (1)\\ \frac{\partial \mathbf{b}}{\partial t} &= \nabla\times(\mathbf{u}\times\mathbf{b}) + \eta_T\,\nabla^2\mathbf{b} \quad (2)\\ \nabla\cdot\mathbf{v}=0 \quad &, \quad \nabla\cdot\mathbf{b}=0\,, \end{aligned}$$

where $\mathbf{v}$ is the fluid velocity, $\mathbf{b}$ is the magnetic field, p is the hydrodynamic pressure, and ν_T and η_T are the eddy viscosity and magnetic resistivity, respectively. Only infinite kinetic and magnetic Reynolds numbers are considered, so that the only dissipative mechanism in the system is the EDQNM-based spectral eddy viscosity and magnetic resistivity.

As shown in [12], the eddy viscosity and resistivity are much larger than the backscatter counterpart, except very close to the cusp $k/k_c \lesssim 1$ where the ratio is still larger than two. Therefore, the effect of backscatter on the total amount of subgrid transfer will be neglected in the present study. To parameterize the eddy viscosity ν_T and magnetic resistivity η_T for use in the LES, these functions are fit as a function of k/k_c from the numerical values obtained from the EDQNM closure computations [9, 12]. The form used for $\nu^+(k|k_c;t)$ and $\eta^+(k|k_c;t)$ is

$$\begin{aligned} &\nu^+(k|k_c;t) = \\ &\left[0.0023398 + 0.0392187\left(\frac{k}{k_c}\right) + 3.73676\left(\frac{k}{k_c}\right)^4\right] \\ &\times\exp\left[4.67997 - 13.121\left(\frac{k}{k_c}\right) + 6.97447\left(\frac{k}{k_c}\right)^2\right] \\ &\eta^+(k|k_c;t) = \\ &\left[0.00267451 + 0.0509853\left(\frac{k}{k_c}\right) + 3.61643\left(\frac{k}{k_c}\right)^4\right] \\ &\times\exp\left[4.51932 - 13.4592\left(\frac{k}{k_c}\right) + 7.37105\left(\frac{k}{k_c}\right)^2\right] \end{aligned}$$

where k_c is the cutoff wavenumber, and the eddy viscosity and magnetic resistivity are conventionally normalized as

$$\nu_T(k|k_c;t) = \nu^+(k|k_c;t)\sqrt{\frac{E_K(k_c,t)}{k_c}} \quad (3)$$

$$\eta_T(k|k_c;t) = \eta^+(k|k_c;t)\sqrt{\frac{E_M(k_c,t)}{k_c}}\,. \quad (4)$$

The kinetic and magnetic energy are

$$E_K(t) = \frac{1}{2}\int_V \mathbf{v}^2\,d^3x \;, \quad (5)$$

$$E_M(t) = \frac{1}{2}\int_V \mathbf{b}^2\,d^3x\,, \quad (6)$$

respectively, with V the volume of the domain. The kinetic to magnetic energy (Alfvén) ratio $r_A = E_K(t)/E_M(t)$ characterizes the energy ratio at the large scales of the flow. The kinetic and magnetic enstrophies are

$$\Omega_K(t) = \frac{1}{2}\int_V (\nabla\times\mathbf{v})^2\,d^3x \;, \quad (7)$$

$$\Omega_M(t) = \frac{1}{2}\int_V (\nabla\times\mathbf{b})^2\,d^3x\,, \quad (8)$$

respectively. The kinetic to magnetic enstrophy ratio $r_B = \Omega_K(t)/\Omega_M(t)$ characterizes the energy ratio at the small scales of the flow.

A three-dimensional freely-decaying LES is performed in a $(2\pi)^3$ periodic computational domain using uniform $N^3 = 128^3$ and $N^3 = 64^3$ computational grids. The dissipative terms in the kinetic and magnetic equations are given respectively by (3) and (4).

The initial fields are constructed as a sum of large-scale Fourier modes with random amplitudes and phases. The kinetic and magnetic energy are both normalized to 0.5 ($r_A = 1$) and the initial cross-correlation $E_c = 2\langle|\mathbf{u}\cdot\mathbf{b}|\rangle/\langle\mathbf{u}^2+\mathbf{b}^2\rangle$ is approximately 0.2.

The simulation is performed using a pseudospectral code to solve the incompressible three-dimensional equations (1)–(2) with desaliasing performed according to the 2/3 rule. A third-order Runge-Kutta scheme is used for the time integration with a constant timestep $\Delta t = 2.5\times10^{-3}$.

The temporal evolution of the kinetic and magnetic energy exhibits a self-similar decay between $t = 5$ and 18 as shown in Fig. 2. During this period, the kinetic energy decreases slightly more rapidly according to $E_K(t) \sim t^{-1.4}$ than the magnetic energy $E_M(t) \sim t^{-1.3}$. Figures (5) and (6) show the kinetic and magnetic spectra

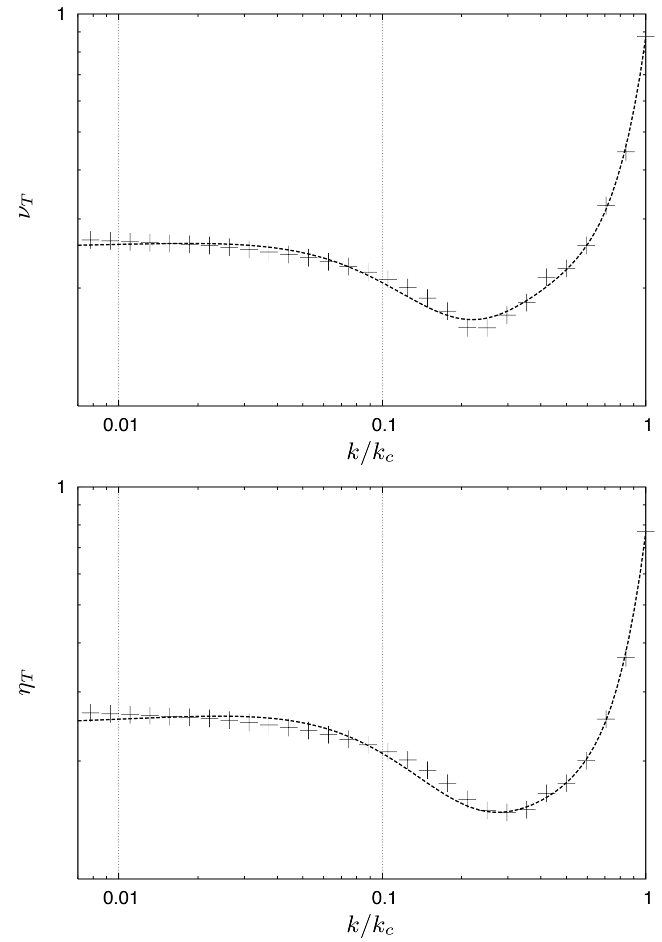

FIG. 1: The eddy viscosity (top) and magnetic resistivity (bottom): fit (–) and values from EDQNM closure calculation (+).

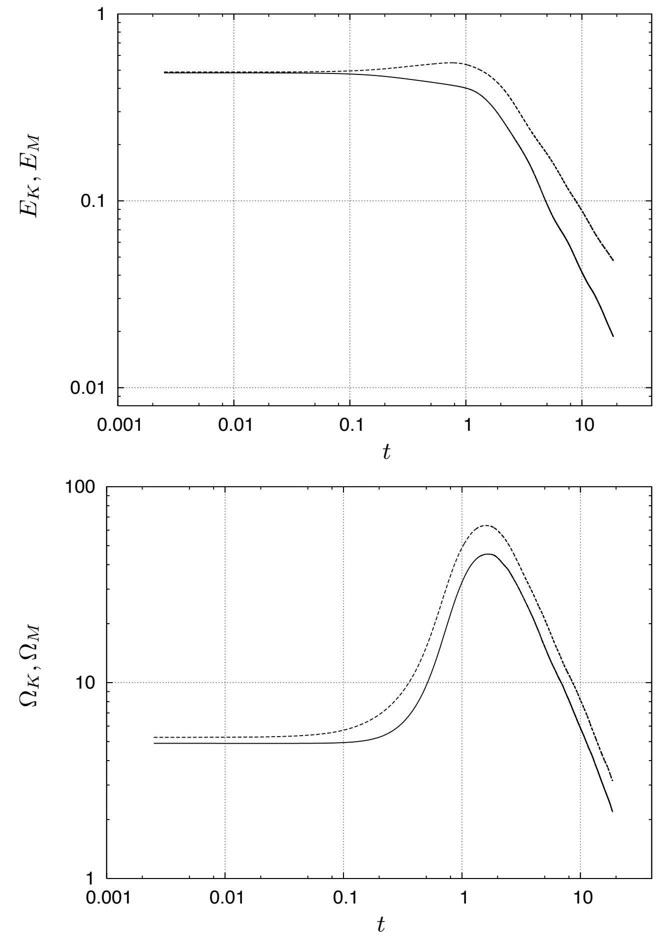

FIG. 2: Temporal evolution of the kinetic (–) and magnetic (−−) energy (top) and of the enstrophy (bottom) between $t = 0$ and 18.

at times $t = 7.5$, 8.5, 9.5, 10.5, 11.5 and 12.5. An inertial subrange with slope $-5/3$ is observed for both spectra, consistent with the EDQNM model [12]. The kinetic and magnetic spectra are shown at different times to show that the simulations are well converged. Between $t = 0$ and 18, the large-scale ratio r_A decreases from 1 to 0.4, whereas the small-scale ratio r_B is stabilized near 0.7 between $t = 2.5$ and 18. This ratio variation in the freely-decaying turbulent case shows that the EDQNM-based LES method could possibly be improved by considering eddy viscosities and magnetic resistivities depending on the Alfvén ratio even if the small-scale ratio r_B remains approximately constant during the course of the simulation.

The spectral subgrid-scale eddy viscosity and resisitivity obtained from EDQNM closure calculations [12] have been parameterized and used to perform LES of three-dimensional, isotropic, non-helical, incompressible, magnetohydrodynamic turbulent flow for quasi-equipartition of energy, *i.e.* $r_A \sim 1$. This subgrid-scale model is robust, as the LES predict $-5/3$ inertial subrange kinetic

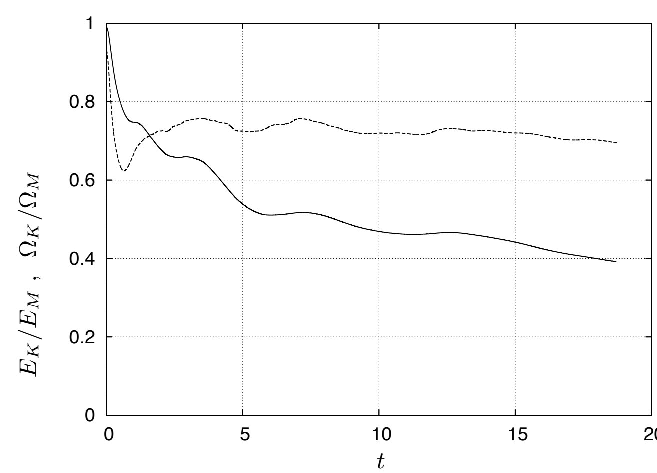

FIG. 3: Temporal evolution of the kinetic to magnetic energy ratio r_A (–) and of the kinetic to magnetic enstrophy ratio r_B (−−) between $t = 0$ and 18.

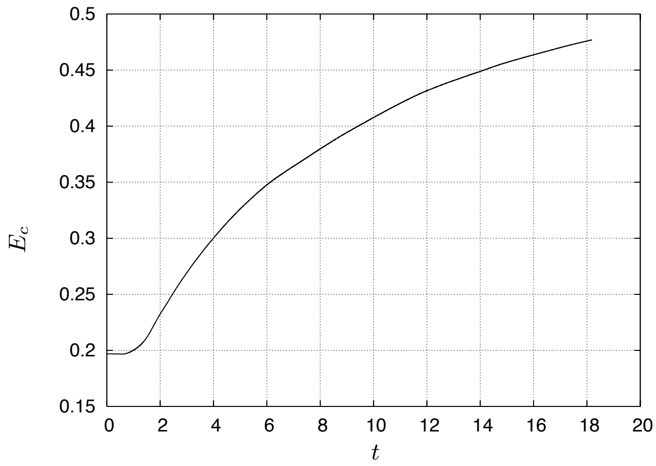

FIG. 4: Temporal evolution of the correlation $E_c = 2\langle|\mathbf{u}\cdot\mathbf{b}|\rangle/\langle\mathbf{u}^2+\mathbf{b}^2\rangle$ between $t=0$ and 18.

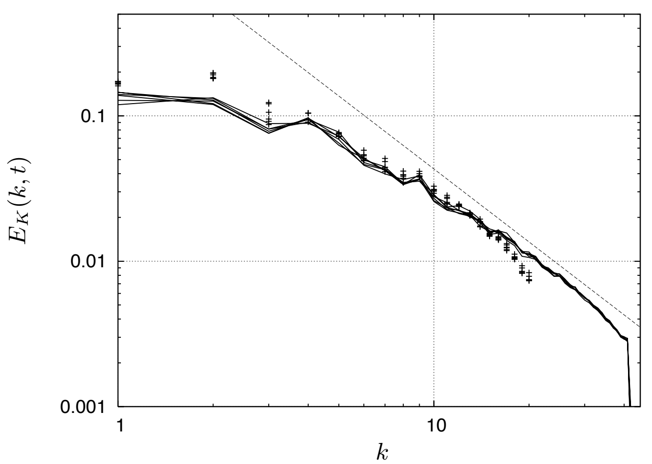

FIG. 5: Log-log plot of the kinetic energy spectrum $E_K(k,t)$ normalized by $E_K(t)$: $N^3 = 128^3$ (–) and $N^3 = 64$ (+).

and magnetic energy spectra, even when the Alfvén ratio decreases from 1 to 0.4 as shown in the simulations.

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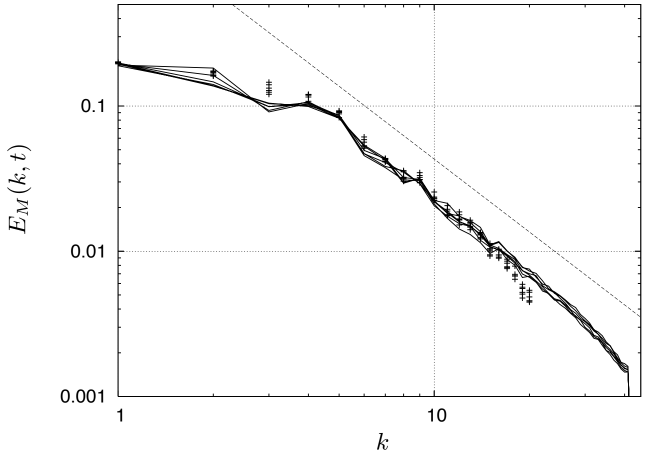

FIG. 6: Log-log plot of the magnetic energy spectrum $E_M(k,t)$ normalized by $E_M(t)$): $N^3 = 128^3$ (–) and $N^3 = 64$ (+).

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